On the available energy of an axisymmetric vortex

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Abstract

A theory of available energy for axisymmetric circulations is presented. The theory is a generalization of the classical theory of available potential energy, in that it accounts for both thermal and angular momentum constraints on the circulation. The generalization relies on the Hamiltonian structure of the (conservative) dynamics, is exact at finite amplitude, and has a local form. Application of the theory is presented for the case of an axisymmetric vortex on an f-plane in the context of the Boussinesq equations.

1 Introduction

This paper applies the concept of pseudoenergy to define an available energy for symmetric circulations in the case of an axisymmetric, stratified vortex under the approximation of f-plane Boussinesq flow. The analysis follows closely the study of the available energy for a (parallel) symmetric circulation presented in (CODOBAN and SHEPHERD, 2003).

Our goal is to derive an expression for the available energy (AE) of the cross-stream circulation, and of the associated energy budget including effects of forcing and dissipation.

The traditional (LORENZ, 1955) AE definition takes into account the conservation of mass and potential temperature (entropy), but not of angular momentum. As Lorenz pointed out himself, one may imagine a situation in which the momentum and mass distributions are in a dynamically stable equilibrium. For such a configuration no energy is available for release to kinetic energy of the cross-stream motion, although in this case by the traditional diagnostic the AE is non zero. It is therefore desirable to redefine the AE in such a way that the coupling of mass and momentum distributions is accounted for, and that for a setup as described above the AE for the cross-stream motion is zero. Mass-momentum coupling is important for practical applications. For example, (SMITH, 1980) has shown that the thermal wind balance of the mass and angular momentum fields constrains the thermally indirect circulation inside the vortex eye, thereby influencing the pressure drop in the eye. In turn, the maximum pressure drop in the eye is related to the maximum wind of the hurricane (EMANUEL, 1997).

An AE diagnostic for axisymmetric flow which includes momentum constraints has been developed at small-amplitude by (VAN MIEGHEM, 1956). To our knowledge, at finite-amplitude such results are not yet available.

The goal of this work is to derive an AE diagnostic for axisymmetric Boussinesq flow, which includes the angular momentum conservation constraint on the cross-stream circulation. Both small- and finite-amplitude expressions are derived. Conditions for dynamical stability of the reference state (RS) are also obtained, as they arise naturally in the framework of the energy-Casimir method employed.

We demonstrate the utility of the theory by showing that for mechanically forced, thermally damped circulations the Lorenz diagnostic of AE may infer the wrong causality; however, by choosing a reference state which properly accounts for the mass-momentum coupling, the correct causality is recovered.

2 Governing equations

We consider axisymmetric flow of an incompressible fluid on an f-plane. We work in the Boussinesq approx-

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