Weak constraints in four-dimensional variational data assimilation

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(Manuscript received March 8, 2007; in revised form September 11, 2007; accepted October 19, 2007)

Abstract

The formulation of four-dimensional variational data assimilation allows the incorporation of constraints into the cost function which need only be weakly satisfied. In this paper we investigate the value of imposing conservation properties as weak constraints. Using the example of the two-body problem of celestial mechanics we compare weak constraints based on conservation laws with a constraint on the background state. We show how the imposition of conservation-based weak constraints changes the nature of the gradient equation. Assimilation experiments demonstrate how this can add extra information to the assimilation process, even when the underlying numerical model is conserving.

Zusammenfassung


1 Introduction

Data assimilation for numerical weather prediction (NWP) aims to use information from observations together with a numerical forecasting model to produce the best estimate of the state of the atmosphere. Many operational forecasting centres have moved towards variational assimilation techniques, in which the analysis is found by minimizing a function measuring the distance between observations and the model state. However since the observations themselves are insufficient to determine the atmospheric state, extra constraints must be incorporated into the variational problem. Such constraints fall into two categories, strong constraints, which must be satisfied exactly, and weak constraints, which need be only approximately satisfied (Sasaki, 1970). Of the first type the most common approach is to incorporate the numerical model as a strong constraint, so that a sequence of states over a time interval of observations must satisfy the model equations. This leads to the method of four-dimensional variational assimilation (4D-Var), which is now operational in many forecasting centres (Rabier et al., 2000; Larocche et al., 2005; Rawlins, 2005). Of the weak constraints the most common is the requirement that the analysed state be close to a previous short-range forecast or background field. This leads to the so-called ‘background term’ within the 4D-Var cost function and such a term is now standard in almost all implementations of 4D-Var.

Other constraints may also be added into the cost function in two ways. The most common way is the incorporation of linear constraints through the formulation of the background error covariance matrix. This method is particularly used to enforce balance conditions within the analysis. Such constraints are specified in the assimilation system by means of a variable transformation, which is designed implicitly to enforce relationships between the analysis increments (Parrish and Derber, 1992; Cullen, 2003). Alternatively weak constraints may be added as extra terms on the cost function, to enforce balance or other conditions only weakly. This method has been used to filter high frequency waves from the analysis or to smooth the final solution (Gauthier and Thépaut, 2001; Lin et al., 2002). Other weak constraints can be used to allow for the fact that the model itself is not perfect (Zupanski, 1997; Nichols, 2003; Trémolet, 2006).

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0941-2948/2007/0249 $ 4.50